# 3.6 WORD PROBLEMS LEADING TO QUADRATIC EQUATIONS

**Worked Example**
The sum of two numbers is 18. The sum of the squares of the numbers is 194. Find the two numbers.

**Solution:**
Let one of the numbers be $x$, then the other is $(18 - x)$. The sum of their squares is $x^2 + (18 - x)^2$. This sum is equal to 194.

$$\begin{aligned} x^2 + (18 - x)^2 &= 194 \\ x^2 + 324 - 36x + x^2 &= 194 \\ 2x^2 - 36x + 324 &= 194 \\ 2x^2 - 36x + 130 &= 0 \\ x^2 - 18x + 65 &= 0 \\ (x - 5)(x - 13) &= 0 \\ x - 5 = 0 \quad &\text{or} \quad x - 13 = 0 \\ x = 5 \quad &\text{or} \quad x = 13 \end{aligned}$$

| | | |
|---|---|---|
| $x$ | $-5$ | $-5x$ |
| $x$ | $-13$ | $-13x$ |
| | | $-18x$ |

If we take $x = 5$, then the other number is $(18 - 5)$, i.e. 13.
If we take $x = 13$, then the other number is $(18 - 13)$, i.e. 5.
Therefore the two numbers are 5 and 13.

**MATHSTORY**

*Many applied problems can be translated into a formula or statement that two quantities or expressions are equal. There is no shortcut for solving applied problems. A guideline to follow was proposed by George Polya (1887–1985) which uses 4 steps:*

1. *Understand the problem.*
2. *Devise a plan.*
3. *Carry out the plan.*
4. *Look back and check.*

*In essence, to solve an applied problem, we first translate the words of the problem into an algebraic equation and solve the equation.*

## Exercise 3.6

*answers on p. 429*

1. The sum of a number and its square is 156. Find the number.

2. The square of a number is equal to 17 times that number. What is the number?

3. The difference between two numbers is 7. The product of the numbers is 144. Find the smaller number.

4. The area of a rectangle is 84 cm$^2$. If the length is 5 cm longer than the width, find the length of the rectangle.

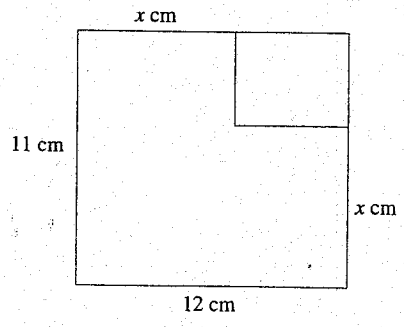
5. I think of a number and add 7 to it, then I multiply the sum by the original number and the result is 60. Find the number.

6. The square of a number is smaller than 12 times the number by 32. Find the number.

7. The area of a triangle is 24 cm$^2$. If its height is 2 cm longer than its base, find the base of the triangle.

**8.** The sum of the square of two consecutive odd numbers is 290. Find the two numbers.

**9.** The area of the shaded part is 112 $cm^2$. Find the value of $x$.




***10.** A farmer encloses a rectangular piece of land which has an area of 2 800 $m^2$ with a fence 220 m long. Find the length and the width of the piece of land.

***11.** A rectangular lawn, 36 m long and 15 m wide, has a path of uniform width around it. If the area of the path is 910 $m^2$, find the width of the path.

***12.** Mr Yang's daily wage was \$5 more than Mr Chen's. Although Mr Yang worked 3 days less than Mr Chen, they earned \$180 each. Find the number of days Mr Yang worked.

***13.** A car travelling uniformly covered 120 km from town $A$ to town $B$. It would have saved 24 min, had it travelled 15 km/h faster. Find the speed of the car.

***14.** A man bought a number of pens for \$63. If the cost per pen was reduced by \$1, he would get 1 more pen by paying \$1 more. Find the cost of each pen.

## Chapter Review

1. A **literal equation** is one in which letters are used to replace the numerical coefficients and the constant terms.

   *Example:* $(ax + b) - cx = 0$

2. A **formula** may be transformed by making a specified variable as the subject of the formula.

   *Example:*

   $A = \frac{1}{2}h(a + b)$ can be transformed to $b = \frac{2A}{h} - a$ by making $b$ the subject of the formula.

(p) $(2x + 3)(3x - 2)$ (q) $(1 - 2x)(1 + 5x)$
(r) $(2x + 1)(4x + 2)$ or $2(2x + 1)(2x + 1)$
(s) $(6x + 1)(x - 3)$
(t) $(2x - 4)(6x + 3)$ or $6(x - 2)(2x + 1)$
(u) $(2x - 2)(x + 5)$ or $2(x - 1)(x + 5)$
(v) $(6x + 2)(2x - 6)$ or $4(3x + 1)(x - 3)$
(w) $(2x + 8)(3x - 2)$ or $2(x + 4)(3x - 2)$
(x) $(x + 1)(3x + 3)$ or $3(x + 1)(x + 1)$

### Exercise 3.5 *(p. 65)*

**1.** (a) $x = 0$ or $\frac{7}{3}$ (b) $x = 0$ or $-\frac{1}{2}$
(c) $x = 0$ or $-\frac{5}{4}$ (d) $x = 6$ or $-6$
(e) $x = \frac{7}{2}$ or $-\frac{7}{2}$ (f) $c = 3$ or $-1\frac{1}{2}$
(g) $d = -\frac{4}{3}, 2$ (h) $x = -\frac{1}{3}, 3$
(i) $p = -4, \frac{2}{3}$ (j) $a = -\frac{5}{4}$ or $-\frac{1}{7}$
(k) $x = 3$ or $-3$ (l) $y = -\frac{9}{4}$ or $-\frac{7}{3}$
(m) $y = -50$ or $-60$ (n) $p = -\frac{13}{3}$ or $-7$
(o) $c = \frac{1}{2}$ or $-2$ (p) $a = 6$ (repeated)
(q) $y = -\frac{1}{5}$ (repeated) (r) $x = 2$ or $3$
(s) $x = \frac{2}{3}$ or $3$ (t) $x = \frac{11}{3}$ or $-1$
(u) $x = \frac{3}{2}$ (repeated) (v) $x = -3$ or $-\frac{1}{3}$
(w) $x = 1$ or $\frac{1}{5}$ (x) $x = 11$ or $\frac{1}{7}$
(y) $x = 1$ or $\frac{2}{3}$ (z) $x = 1$ (repeated)

**2.** (a) $x = -3$ or $4$ (b) $x = -\frac{3}{2}$ or $\frac{10}{3}$
(c) $x = 2$ or $-1$ (d) $x = -\frac{4}{5}$ or $\frac{5}{3}$
(e) $x = 1$ or $\frac{9}{5}$ (f) $x = 1$ or $-2$
(g) $x = -\frac{3}{2}$ or $\frac{1}{3}$ (h) $x = -\frac{2}{3}$ or $1$
(i) $x = 1$ or $-\frac{2}{3}$ (j) $x = \frac{5}{2}$ (repeated)

### Exercise 3.6 *(p. 66)*

**1.** 12 or $-13$ **2.** 17 or 0 **3.** $-16$
**4.** 12 cm **5.** 5 or $-12$ **6.** 8 or 4
**7.** 6 cm **8.** 11, 13 **9.** 7
**10.** 70 m, 40 m **11.** 7 m **12.** 9 days
**13.** 60 km/h **14.** \$9

## Chapter 4

### Exercise 4.1 *(p. 76)*

**1.** \$5 257.20 **2.** \$126
**3.** \$684 **4.** 38 *l*, \$39.90
**5.** 137.5 km **6.** 600 km/h
**7.** 17 00
**8.** (a) $1\frac{1}{2}$ km/min (b) 25 m/s
**9.** (a) 275 min (b) 90
**10.** (a) $42\frac{1}{12}$ h (b) \$319.83
**11.** (a) 07 31 (b) 2 km
**12.** (a) 9 min (b) 12 km/h
(c) 7.5 km/h
**13.** (a) $1.5 \times 10^{11}$ km (b) $2.59 \times 10^3$ s
**14.** (a) \$96 (b) $3\frac{1}{4}$ h
(c) \$16 (d) \$103.55
**15.** (a) \$1 260 (b) \$650
**16.** 27 km
**17.** (a) 250 km (b) 3 h $7\frac{1}{2}$ min
**18.** (a) 100 km/h (b) 16 00
**19.** \$36, \$18, \$9 **20.** *A*: \$33, *B*: \$55
**21.** \$250, \$350, \$450 **22.** \$8 000, \$4 000, \$2 000
**23.** 2 h 30 min
**24.** (a) \$952 (b) 9 days
**25.** \$18
**26.** (a) 9.3 kg (b) \$330
**27.** 3 h **28.** $7\frac{1}{2}$ days
**29.** (a) 282 000 (b) 340 000
**30.** (a) 150 (b) 85%
**31.** (a) 15 cm (b) 324 $cm^2$
**32.** 15% **33.** 576
**34.** 22.2% **35.** (a) 35% (b) 36
**36.** 240 m*l* **37.** 30 g
**38.** 500 kg, 1 000 kg **39.** 12 %
**40.** 200

### Exercise 4.2 *(p. 85)*

**1.** \$91.00 **2.** 25% **3.** 81 cents
**4.** 12% **5.** Gain 20% **6.** \$80
**7.** \$220 **8.** 25% **9.** 20%
**10.** \$20.40 **11.** \$138